Narrowband Digital Modulation

EE 233B

Wireless Communication Systems

Overview

- ***Choice of modulation technique is an important consideration***
  - ***Main considerations***
    - *Spectrum efficiency (bps/Hz)*
    - *Power efficiency (SNR for a given BER)*
    - *Complexity/cost/power consumption*
    - *Robustness to Impairments*
      - *Linear distortion (filters)*
      - *Nonlinear distortion (amplifiers)*
      - *Interference (ACI, CCI, ISI)*
      - *Radio propagation (path loss, fading, doppler, delay spread)*
- ***Digital vs. analog modulation***
    - *Better spectral efficiency*
    - *Resistance to channel impariments*
    - *Lower power*
    - *Better security and Privacy*

Overview cont'd

- ***In general any modulated signal can be represented as***
  - $s(t) = A(t)\cos(w_c t + \phi(t)) = Re\{A(t)\, e^{\phi(t)}\, e^{jw_c t}\}$
    - *A(t) amplitude* $\phi(t)$*: pahse* $d\phi(t)/dt$ *: instantaneous frequency*
  - $s(t) = A(t)\cos(\phi(t))\cos(w_c t) - A(t)\sin(\phi(t))\sin(w_c t)$
    - $A(t)\cos(\phi(t))$ *: In-phase component*
    - $A(t)\sin(\phi(t))$ *: Quadrature component*
  - ***In general it is assumed that the variations in phase and amplitude are much slower than the carrier frequency***
- ***Noise***
  - ***Assume that additive white Gaussian noise (AWGN) is filtered in the receiver resulting in narrowband noise n(t)***
    - $n(t) = n_I(t)\cos(w_c t) - n_Q(t)\sin(w_c t)$
- *Two types of digital modulation*
  - *Linear*
  - *Constant envelope*
    - *More bandwidth*
    - *Resistant to changes in signal amplitude caused by the channel*

Modulation Schemes in Wireless Systems

Modulation Schemes in Different Wireless Systems

Systems	Modulation Scheme Voice	Modulation Scheme Data	Comments
Analog cellular			
AMPS (U.S.), TACS (U.K.)	FM	FSK	AMPS: R = 10 Kbps, spectral efficiency = 0.33 bits/sec/Hz
NTT (Japan)	FM	FSK	
MATS-E (German)	PM	FFSK	
Nordic 450/900	PM	FSK (MSK)	
C-450 (German)	PM	FSK	
Digital cellular			
GSM	GMSK	GMSK	R_d = 270.8 Kbps, spectral efficiency = 1.35 bits/sec/Hz
NADC (IS-54)	$\pi/4$-DQPSK	$\pi/4$-DQPSK	R_d = 48.6 Kbps, spectral efficiency = 1.62 bits/sec/Hz
JDC	$\pi/4$-DQPSK	$\pi/4$-DQPSK	R_d = 42 Kbps, spectral efficiency = 1.6 bits/sec/Hz, B_bT = 0.3
Cordless Telephone			
CT1	Analog, FM	---	
CT2	Digital, MSK	MSK	R_d = 72 Kbps, spectral efficiency = 0.72 bits/sec/Hz
CT3, DECT	GMSK	GMSK	R_d = 1.152 Mbps, spectral efficiency = 0.67 bits/sec/Hz, B_bT = 0.5

Linear Modulation

- $S(t) = Re\{ u(t) e^{jw_c t} \}$
  - s(t): transmitted signal   u(t): baseband equivalent signal
  - $u(t) = \Sigma_n d_n g(t-nT)$ $d_n = a_n + jb_n$
    - T: symbol period
    - d_n: a sequence of complex numbers representing the information sequence
- Using this representation, all linear modulation schemes can be represented as:
  - $s(t) = [\Sigma_n a_n g(t-nT)] \cos(w_c t) - [\Sigma_n b_n g(t-nT)] \sin(w_c t)$
- Types of linear modulation
  - M-arry Quadrature Amplitude Modulation (M-QAM)
    - ***Square constellation***
  - M-arry phase shift keying (M-PSK)
    - 4-PSK (QPSK) same as 4-QAM
      - offset QPSK π/4-QPSK
  - Orthogonal Frequency Division Multiplexing (OFDM)

Passband Modulation

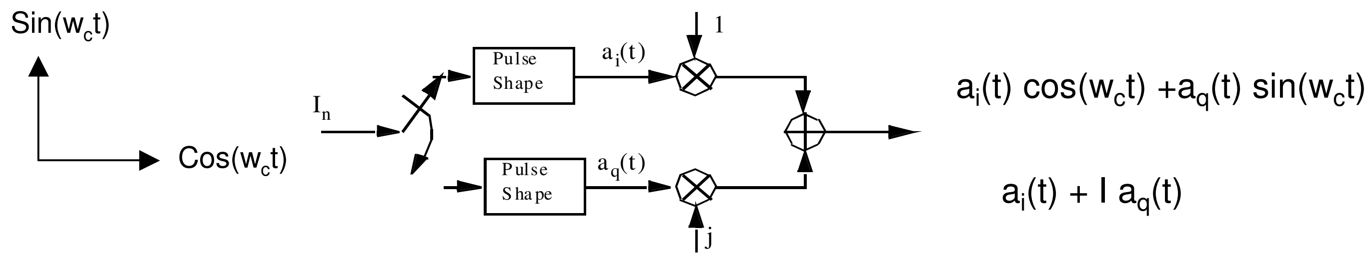
- ***Frequency content is centered around a carrier frequency f_c***
  - ***Requires carrier as well as timing adjustment at the receiver***
  - ***Two dimensional and multi-dimensional transmission using sine and cosine waveforms***
- ***Allows frequency division multiplexing (FDM)***
  - ***Frequency division multiple access (FDMA)***
- ***Examples of passband modulation***
  - ***Quadrature Amplitude Modulation (QAM)***
  - ***Phase Shift Keying (PSK)***
  - ***Frequency Shift Keying (FSK) and GMSK***
  - ***Orthogonal Frequency Division Multiplexing (OFDM)***

Aside: Baseband Equivalent Signals and Systems

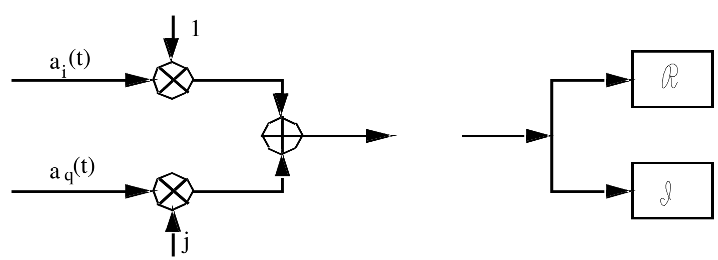
- ***All Simulations take place in the digital domain***
  - ***Nyquist Theorem: Simulation sampling frequency has to be at least twice the highest frequency content of the signal***
- ***For passband signals***
  - ***Carrier frequency can range from a few tens of kHz to GHz***
  - ***Sampling at twice the carrier frequency requires a large amount of unnecessary oversampling to model the carrier which is "uninteresting"***
- ***Baseband euivalent signal models allow us to model the narrowband passband signal at baseband using complex notation***
  - ***Simulation sampling frequency is reduced to two times the signal bandwidth instead of the carrier frequency***
  - ***Significant speed improvement***

Baseband Equivalence, A Heuristic Approach

- Sine and Cosine signals form a 2-dimensional orthogonal basis
- $\mathcal{R}$ and $\mathcal{I}$ axes form a 2-dimensional orthogonal basis



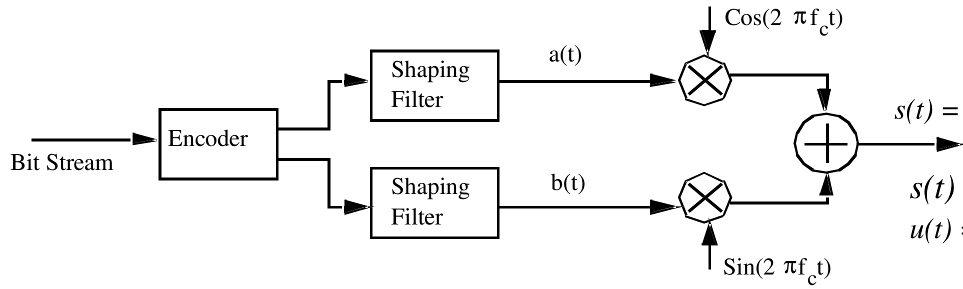
- ***Let $\mathcal{R}$ and $\mathcal{I}$ replace cosine and sine in our studies***
  - ***Transmit $a_i(t)$ using the real component and $a_q(t)$ using the imaginary component of the complex signal***
  - ***Equivalent transmitter and receiver block diagram***



Baseband Equivalence cont'd

- ***The complex equivalent signals occupy a bandwidth equal to the bandwidth of the modulating signals***
  - ***The simulation sampling frequency needs to be twice the bandwidth of the baseband signal not the passband signs***
- ***Baseband Equivalence of Channels***
  - ***Channels by themselves are not necessarily band limited***
  - ***Passband systems use bandpass filters to reduce the amount of noixe entering the system and to reduce inter-channel interference***
  - ***The section of the channel which is of interest lies within the frequency range defined by the bandpass filters***
    - *The channel can be considered to be a narrowband channel centered about a frequency f_c*
  - ***Can define baseband equivalence for channels.***

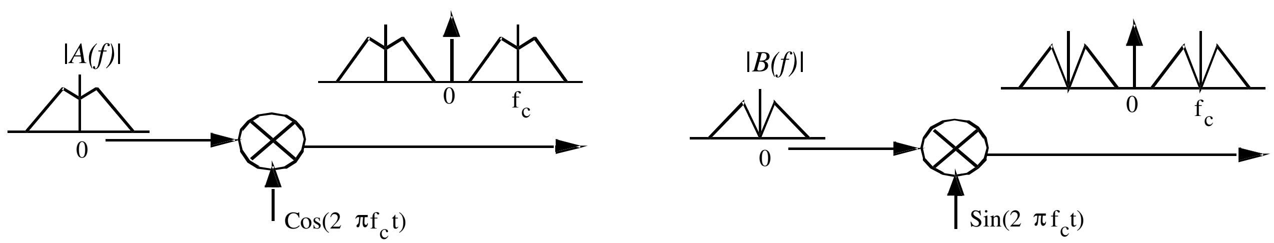
Frequency Domain Representation

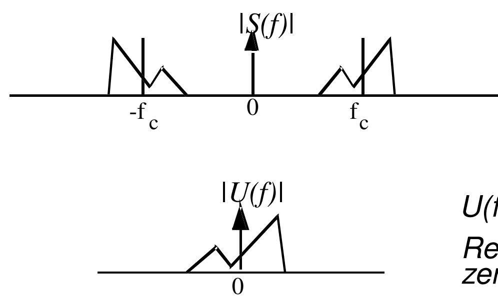


$s(t) = a(t)\cos(2\pi f_c t) + b(t)\sin(2\pi f_c t)$

$s(t) = Re\{u(t)\, e^{j2\pi f c t}\}$

$u(t) = a(t) - jb(t)$

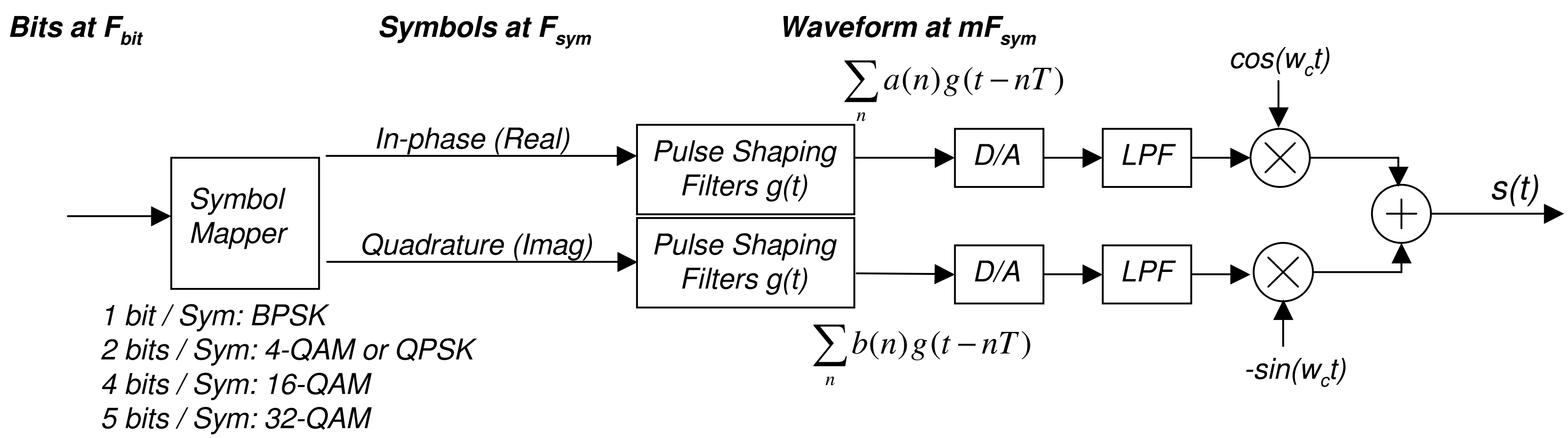




U(f)is not symmetric about zero.

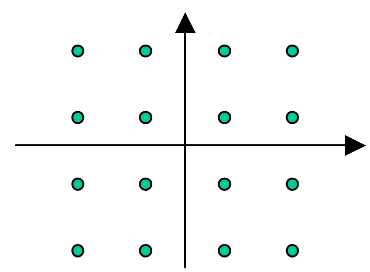
Real signals are always symmetric about zero frequency.

Structure for Generating Linear Modulated Signals



$$s(t) = \sum_n a(n) g(t-nT) \cos(w_c t) + \sum_n b(n) g(t-nT) \sin(w_c t) \qquad T_s = 1/F_{sym}$$

- ***Frequency content of the transmitted signal is depends on the pulse shape g(t)***
- ***Each symbol could correspond to a different phase AND/OR amplitude***



16-QAM Constellation

Phase Shift Keying

- ***Data is transmitted in the phase on the carrier***

$$s(t) = \sum_n \cos(2\pi f_c t + \phi_n)$$

$$s(t) = \sum_n \cos(2\pi f_c t)\cos(\phi_n) - \sin(2\pi f_c t)\sin(\phi_n)$$

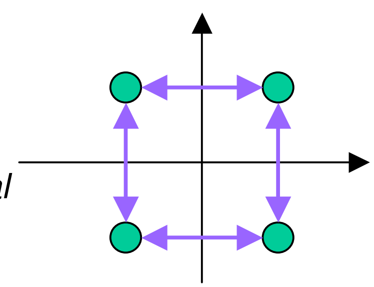
ϕ_n is different for each symbol

- ***The second equation suggests that***
  - ***PSK is similar to QAM***
    - *Replace integer scaling of sine and cosine signals with cos(ϕ_n) and sin(ϕ_n)*
    - *QPSK is identical to 4-QAM*
  - ***Symbols reside on a circle in the complex plane***

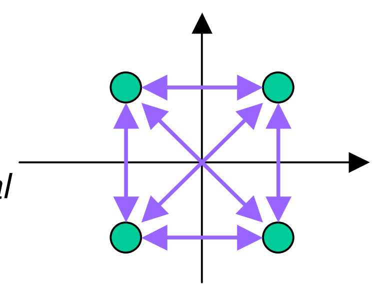
Offset QPSK

- ***Used in uplink of IS-95 (CDMA) 2nd generation cellular systems***
- ***The Inphase (I) and the Quadrature (Q) rails are offset (staggered) by half a symbol period***
  - ***Transitions occur every $T_{sym}/2 = T_{bit}$ but only on one of the rails***
  - ***Tranistions from one symbol to the next will first traverse along the y (or x) axis for the first T_{bit} seconds and then along the x (or y) axis for the second T_{bit} seconds***
- ***Motivation***
  - ***If QPSK is realized using a pulse shape other than a square pulse, the passband signal enelope will not be constant***
    - *Regular QPSK envelope will go through zero during some transitions*

Transitions for an OQPSK modulated signal



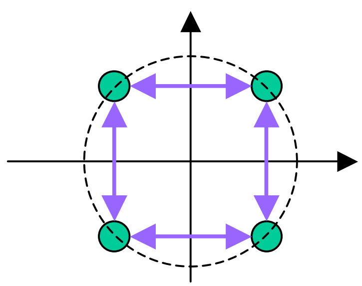
Transitions for a QPSK modulated signal



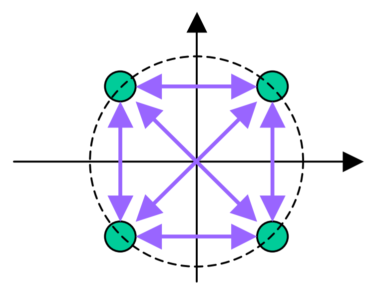
OQPSK cont'd

- ***Regular QPSK envelope will go through zero during some transitions***
  - ***Increased peak-to-average power (PAP) ratio for the modulated signal***
  - ***Requires a more linear front end amplifier to maintain the increased dynamic range***
    - *More DC power is wasted while the signal amplitude is small to guarantee amplifier linearity for periods when the amplitude is large*
    - *Reduced power efficiency*
- ***OQPSK eliminates the need to cross the zero envelope point in going from one symbol to the next***
  - ***Reduced PAP ratio***

OQPSK

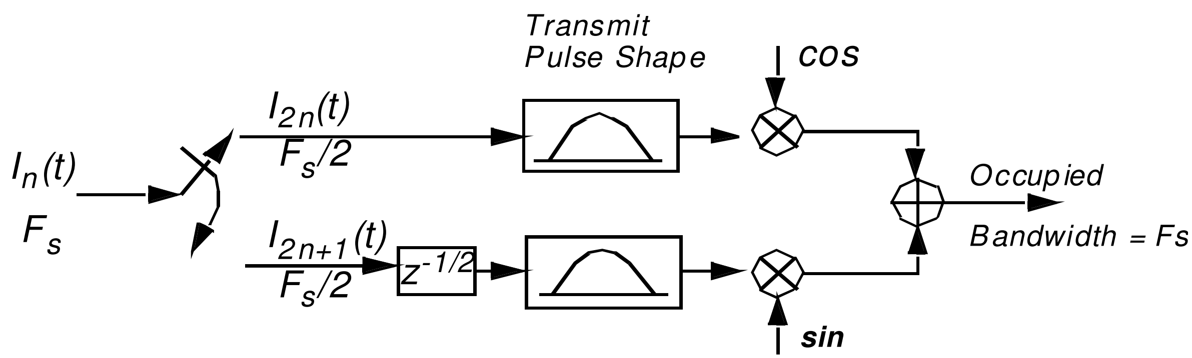


QPSK

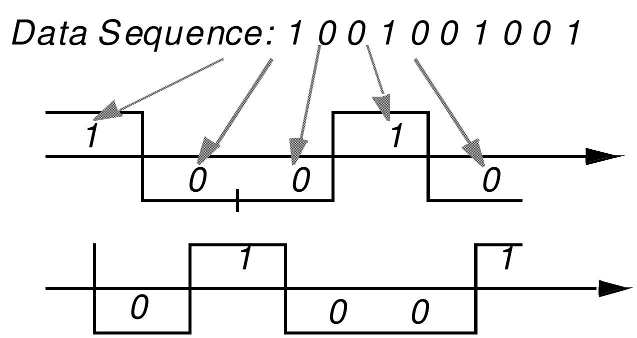


OQPSK Block Diagram

Transmitter block diagram

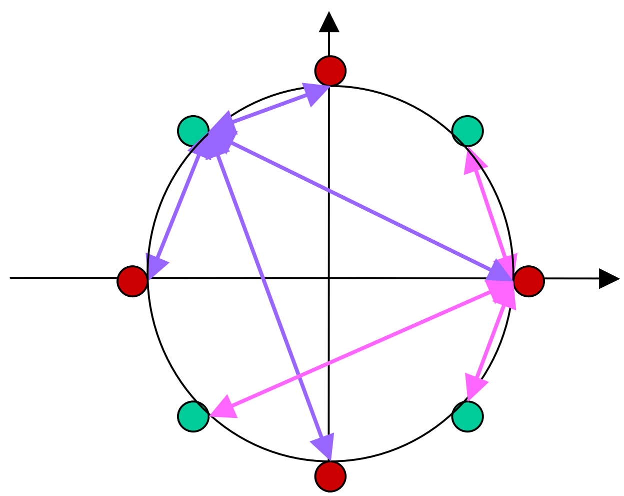


Transmitter waveform



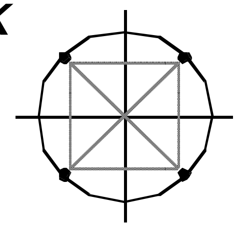
π/4-QPSK

- ***Used in IS-136 North american digital cellular standard***
- ***Use 2 QPSK constellations one rotated by π/4 radians with respect to the other***
  - ***Alternatively pick symbols from one or the other constellation***
- ***Reduces PAP ratio by eliminating signal envelope transitions through the origin of the constellation***

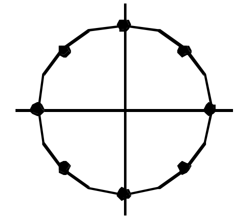


QAM and QPSK Constellations

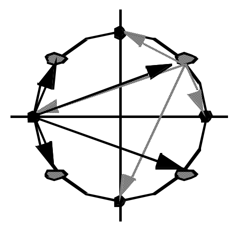
4-QAM, QPSK



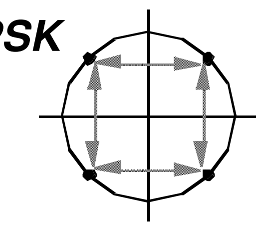
8-PSK



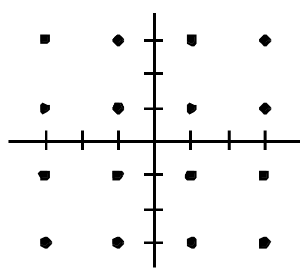
p/***4 QPSK***



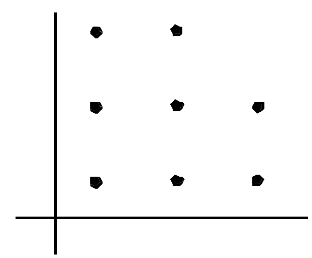
Offset QPSK



16-QAM

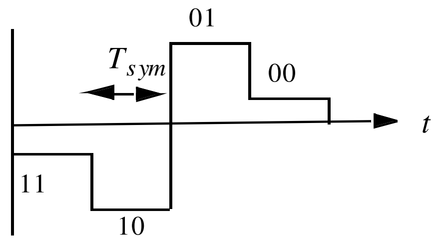


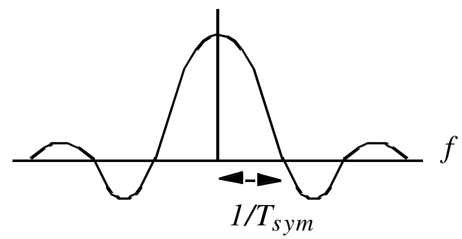
32-QAM



Raised Cosine Pulse Shape

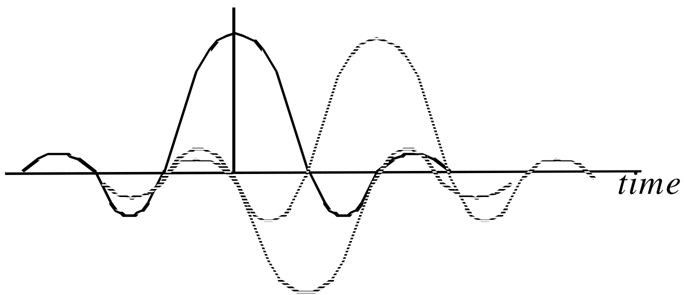
Consider first a square pulse shape

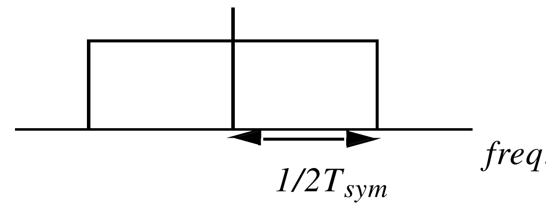




- *Time Domain*
  - *Finite time*
  - *Sampling phase may be off by as much as +/- $T_{sym}/2$ with no ISI*
- *Frequency Domain*
  - *Sinc (sin(x)/x) shape*
  - *Infinite bandwidth*
- *Infinite bandwidth requirement is impractical*

Raised Cosine (cont'd)

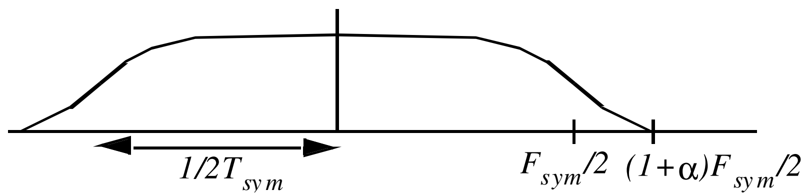




- Time Domain
  - No ISI with perfect timing
  - Sinc has 1/t (1/n) roll-off
  - Infinite ISI (closed eye) with slight timing offset since $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$
- Frequency Domain
  - Ideal (flat) frequency response
  - Band limited

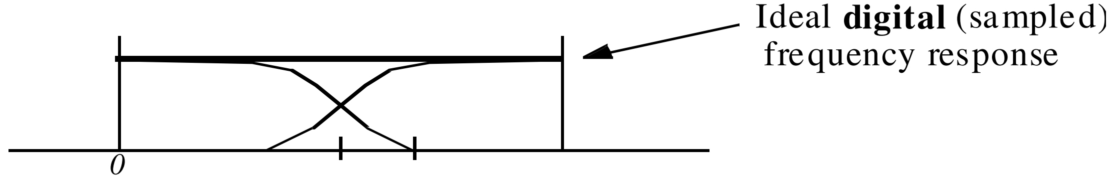
Raised Cosine

- ***Allow controlled frequency roll-off instead of brick wall characteristic***



  – ***The time domain pulse has a sinc-like shape with $1/n^3$ roll off***
    - *Eye remains open even in the presence of sampling phase offset*
    - *α referred to as the roll-off factor (or excess bandwidth) trades off bandwidth for immunity to sampling phase offset*
    - *Larger a results in larger bandwidth occupancy and increased immunity to sampling phase offset*

- ***Consider sampling the raised cosine pulse at the baud frequency***



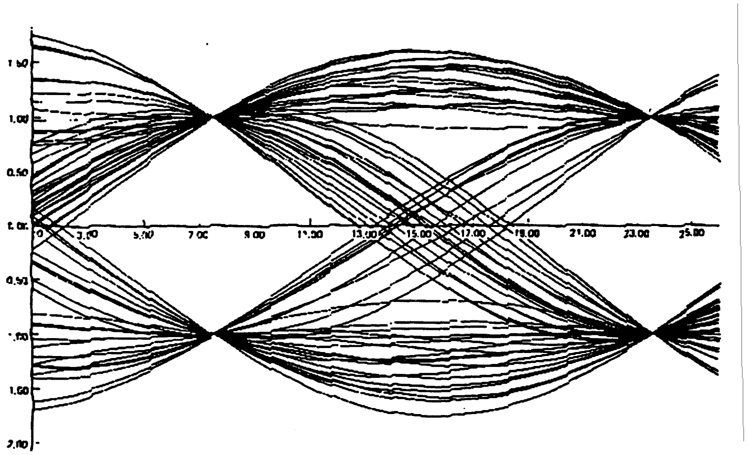
The Raised Cosine Pulse

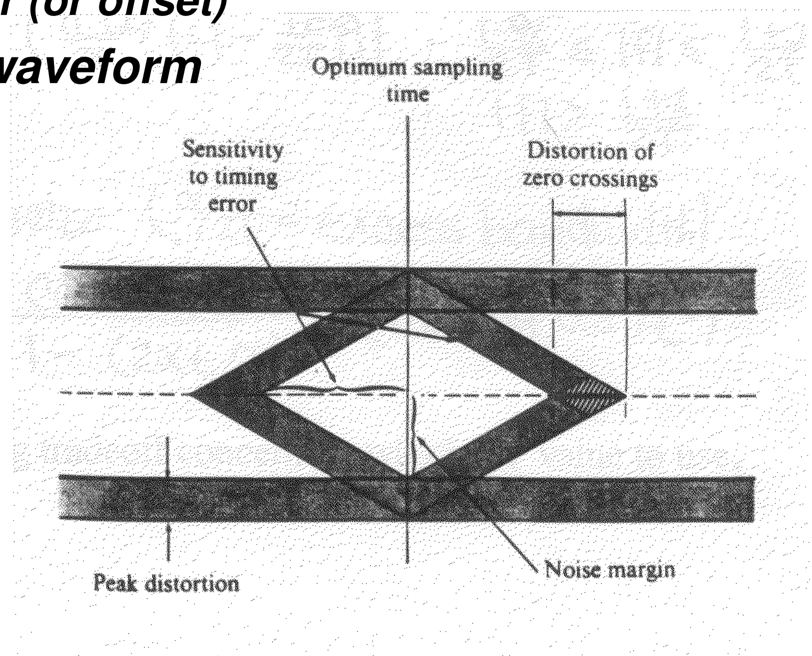
- ***H(f) = 1*** ***|f| < (1-α)/2T***
- ***H(f) = 0.5 + 0.5 sin(πT((0.5T-|f|) / α)*** ***(1-α)/2T < |f| < (1+α)/2T***
- ***H(f) = 0*** ***(1+α)/2T < |f|***

$$h(t) = \frac{1}{T} \frac{\cos(\pi\alpha T}{1 - 4\alpha^2 (t^2 / T^2)} \frac{\sin(\pi t T)}{\pi t / T}$$

Eye Diagram

- *Created by overlaying segments of the received signal*
  - *Segments must be an integer multiple of the symbol period*
- *Eye diagram provides information about*
  - *Optimum sampling instant*
  - *Signal to noise ratio*
  - *Tolerance to sampling phase jitter (or offset)*
- *Eye diagram for a raised-cosine waveform*





Differential QPSK (DQPSK)

- ***In QPSK each symbol represents a unique pair of information bits***
- ***In DQPSK data is differentially encoded***
  - ***Encoded in the phase difference between consecutive samples***
  - ***DQPSK does NOT require carrier phase match between the transmitter and receiver***
- ***QPSK***
  - $s(t) = A \cos(w_c t + (i-1)\pi/2 + \lambda)$
    - *I =1,2,3,4 depending on the desired transmitted symbol*
    - λ*: initial phase*
  - $s(t) = A \cos\phi_i \cos(w_c t) - A \sin\phi_i \sin(w_c t)$
    - *In-phase component* $I_i = \cos\phi_i$
    - *Quadrature component* $Q_i = \sin(\phi_i)$
- ***DQPSK***
  - $I_i = I_{i-1} \cos\Delta\phi_i - Q_{i-1} \sin\Delta\phi_i$ $\qquad Q_i = I_{i-1} \sin\Delta\phi_i + Q_{i-1} \cos\Delta\phi_i$

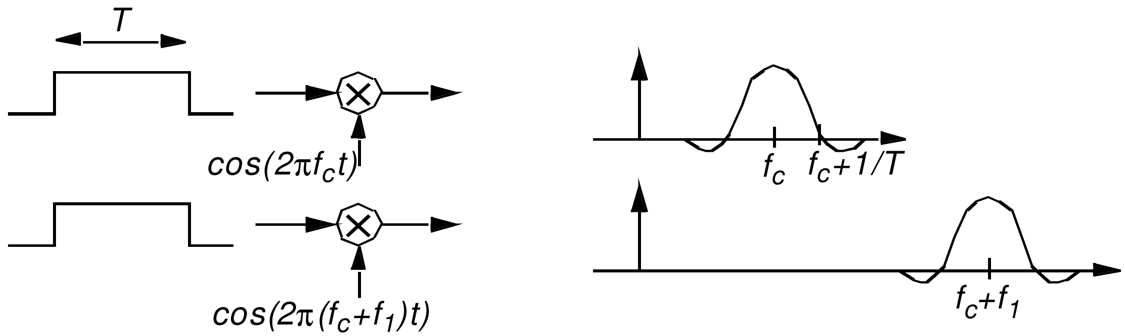
    $\Delta\phi_i = \pi/4, 3\pi/4, -3\pi/4, -\pi/4$ *for symbols 1,2,3,4 respectively*
- ***π/4-DQPSK is the modulation for IS-126/54***

Orthogonal Frequency Division Multiplexing (OFDM) Discrete Multitone (DMT)

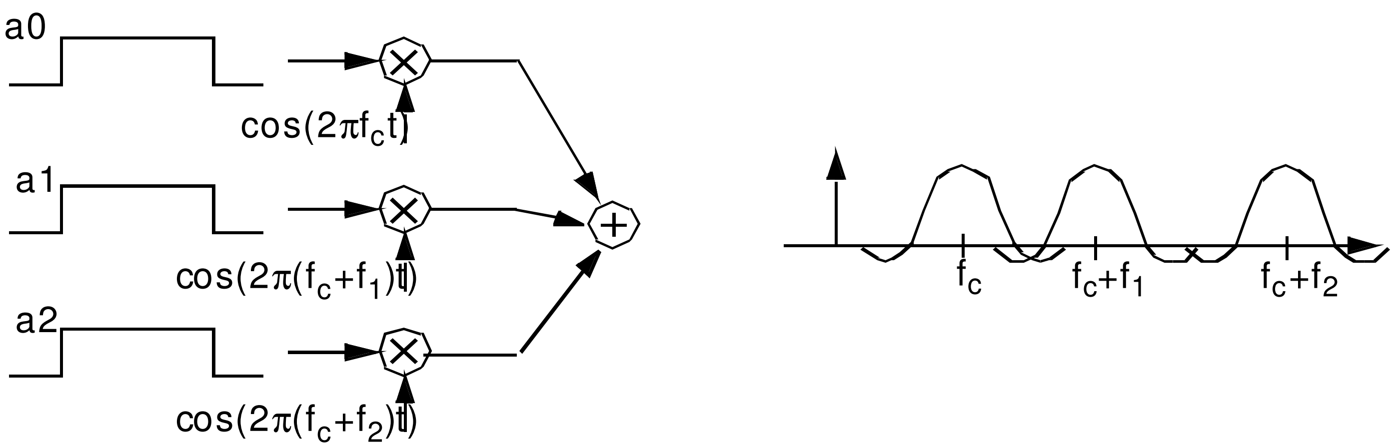
- ***Converts a wideband signal into a series of narrowband signals placed side-by-side in the frequency domain***
- ***Pros***
  - ***Immune to the effects of a dispersive channels***
    - *No need for equalization*
    - *Implemented using an FFT at the transmitter and receiver.*
- ***Cons***
  - ***High peak to average ratio imposes large linearity constraint on transmit power amplifier***
    - *Important consideration in wireless systems*
    - *Some overhead associated with guard interval*
- ***Applications***
  - ***ADSL, European DAB, High speed wireless LANs***

Basic Concept

- ***Consider single frequency modulation***

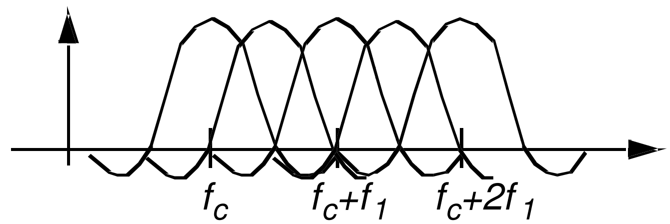


- ***Consider multicarrier modulation***



Orthogonal Frequencies

- ***Let $\Delta f = 1/T$ for multicarrier modulation***
  - ***For each sub-carrier frequency, the contribution from all other sub-carriers are zero***
  - ***Orthogonal waveforms***
  - ***Analogous to the use of sinc pulses in the time domain having brick wall frequency response***



  - ***Orthogonal Frequency Division Multiplexing***
  - ***Each carrier may be modulated independently,***
    - *BPSK, 4- 16- QAM,*
  - ***Frequency selective channels will result in some sub-carriers having higher SNR than others***
    - *Use higher size constellations with higher SNR.*

Realization with an FFT

- ***For each frequency $f_c+n\Delta f$ use sine and cosine (quadrature) waveforms***
  - ***Each quadrature pair is then modulated with a pair of information bits which we will denote as a_n and b_n***
- ***Consider the baseband equivalent signal for each quadrature pair***
  - ***$(a_n+jb_n)\exp(-j2\pi n\Delta ft)$ n=0, ..., N-1 for N pair of sine and cosines***
- ***The output signal y(t) is the sum of quadrature modulated signals***

$$y(t) = \sum_{n=0}^{N-1} (a_n + jb_n) e^{j2\pi n\Delta ft}$$

- ***Let us sample the time signal with a sampling period of (1/NΔf)***

$$y(t)\Big|_{t=mT} = \sum_{n=0}^{N-1} (a_n + jb_n) e^{\frac{j2\pi nm}{N}}$$

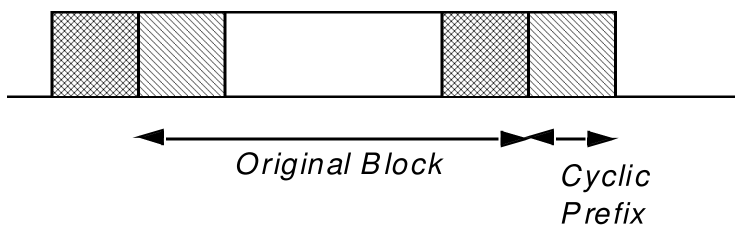
  - ***This is the inverse FFT of the complex sequence (a_n+jb_n)***
    - *Note that the complex baseband notation allows us to represent signals with frequency contents up to the sampling frequency.*

OFDM Transmission

- ***Demodulation***
  - ***The receiver must perform the inverse operation of the modulator***
  - ***FFT at the receiver***
- ***Real Channel suffers from multipath and frequency selectivity (notches in the frequency domain)***
  - ***Multipath***
    - *Causes one block to interfere with another*
    - *Use guard interval between blocks*
  - ***Eliminates the orthogonality among tones***
    - *Use cyclic prefix on each block*
  - ***Frequency selectivity***
    - *Each tone has a different phase rotation and amplitude*

Cyclic Prefix

- ***FFT assumes a cyclic time waveform with period T_{block}***
  - ***Transmitted block can be assumed cyclic***
  - ***Channel multipath is not necessarily cyclic with period Tblock***
  - ***Convolution of channel response and transmitted block is not cyclic***
- ***Cyclic Prefixing the transmitted block can make the received block look cyclic to the FFT***



Minimum Shift Keying (MSK)

- ***A form of continuous phase frequency shift keying (FSK) modulation***
- ***FSK***
  - ***Data bit 0 -> $f_1 = f_c + \Delta f$***
  - ***Data bit 1 -> $f_2 = f_c - \Delta f$***
- ***MSK has modulation index = 1/2***

  $\Delta f = 1/4T$

    - *T is the bit interval*
  - ***Constant amplitude modulation***
  - ***First null of the power spectral density occurs farther out than for QPSK***
  - ***Modulation skirts fall off faster than unfiltered QPSK***
- ***Transmitted signal***

$$s_i(t) = A\cos[2\pi f_i t + \theta_n + \frac{n\pi}{2}(-1)^{i-1}] \qquad i = 1,2$$

MSK

$$s_i(t) = A\cos[2\pi f_c t + 2\pi \Delta f t + \theta_n + \frac{n\pi}{2}(-1)^{i-1}]$$

$$\theta_n = \frac{\pi}{2}\sum_{k=-\infty}^{n-1} I_k \qquad I_k = 1,-1$$

During a bit (symbol) period

$\theta(t) = \theta_o \pm \pi/2\ (t/T_s)$

Phase of signal at the end of odd bit intervals can be +π/2 or -π/2

Phase of signal at the end of even bit intervals can be 0 or 2π

MSK

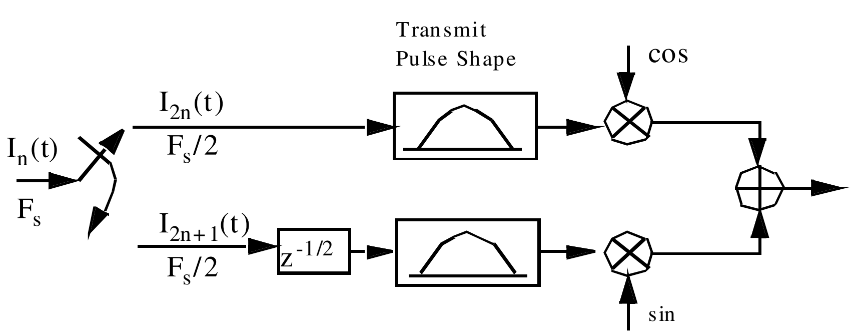
– ***For MSK si(t) can be written in a different form as;***

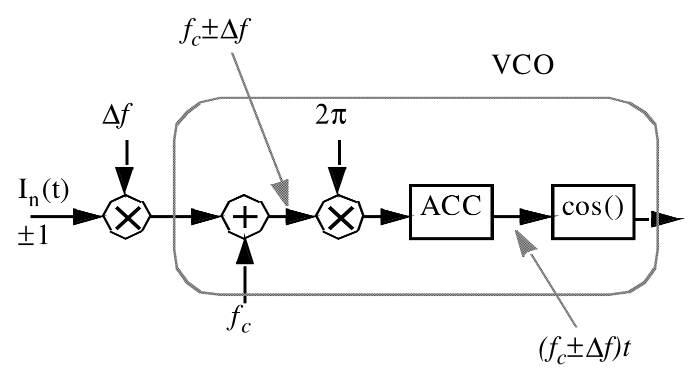
$$s_i(t) = \left[\sum_{n=-\infty}^{\infty} I_{2n} u(t - 2nT) \right] \cos(2\pi f_c t) + \left[\sum_{n=-\infty}^{\infty} I_{2n+1} u(t - 2nT - T) \right] \sin(2\pi f_c t)$$

$$u(t) = \begin{cases} \sin\left(\frac{\pi t}{2T}\right) & 0 \le t \le 2T \\ 0 & otherwise \end{cases}$$

– ***This is offset (staggered) QPSK with a half-sine pulse shape instead of a square, or raised-cosine, etc.***

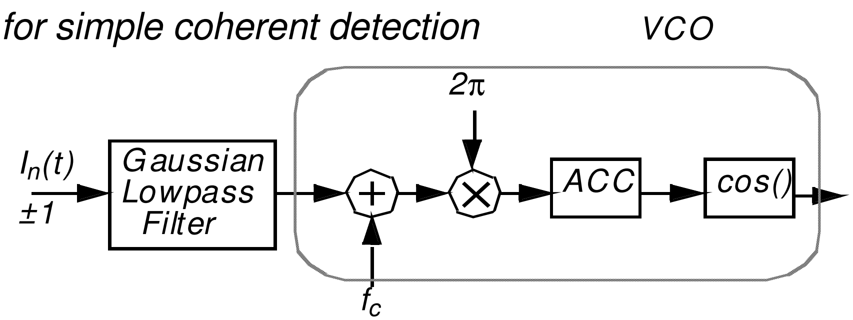
  - *Can be detected using coherent demodulation as with other linear modulation schemes*





Gaussian MSK (GMSK)

- ***Used in GSM, DECT, HIPERLAN***
- ***MSK has considerable out of band radiated energy***
- ***Lower the out of band energy by lowpass filtering the binary input data prior to modulation***
  - ***Guarantees constant envelope property***
- ***Lowpass filter should present the following properties***
  - ***Narrow bandwidth and sharp cutoff***
    - *suppress high frequency components*
  - ***Low overshoot impulse response***
    - *protect against excessive instantaneous frequency deviation*
  - ***Preservation of the filter output pulse area***
    - *Corresponds to a phase shift of $\pi/2$ for simple coherent detection*
  - ***Use a Gaussian Lowpass filter***



Power Spectral Density, Overview

- ***Definition***
  - ***The power spectral density (PSD) gives the distribution of power for a random process over a frequency range of interest***
  - ***The signal energy within a frequency band f1 to f2 is given by the integral of the PSD from f1 to f2.***

$$S_{xx}(f) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-j2\pi f\tau} d\tau$$

    - *$S_{xx}(f)$: PSD of the random process x(t)*
    - *$R_{xx}(\tau)$: Autocorrelation function of x(t)*

$$R_{xx}(\tau) = E\left[x(t)x^*(t-\tau)\right]$$

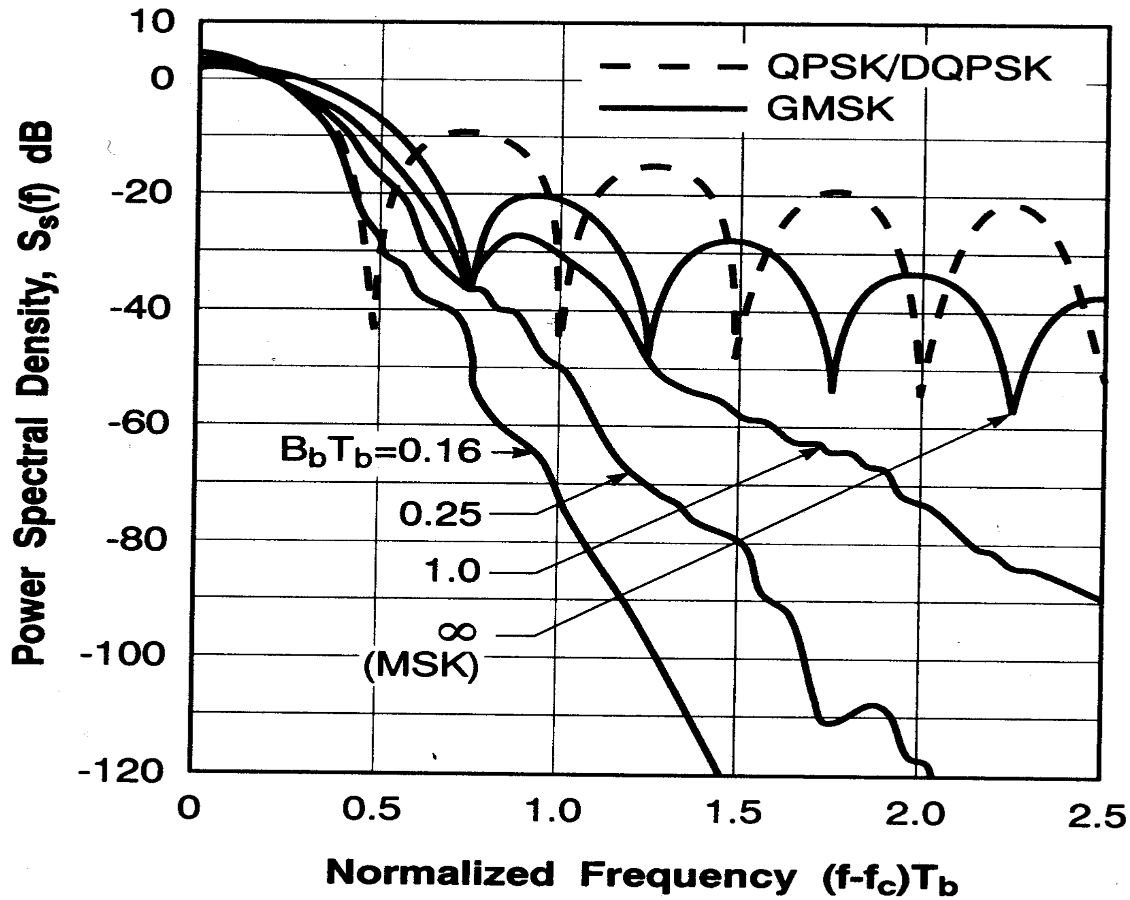
  - ***Recall, for a linearly modulated signal***
    - ***$s(t) = [\, \Sigma_n\, a_n\, g(t-nT)\,]\ \cos(w_c t)\ -\ [\, \Sigma_n\, b_n\, g(t-nT)\,]\ \cos(w_c t)$***

PSD for a Linearly Modulated Signal

- ***Consider a baseband linearly modulated signal***
    - ***$s(t) = \Sigma_n a_n g(t-nT)$***
  - ***$\{a_n\}$ is wide-sense stationary (WSS)***
    - *Mean and autocorrelation functions are constants and independent of time*
    - *WSS is less stringent than strict stationarity since it does not require the pdf to be the same for all time.*
  - ***The information symbols $\{a_n\}$ are mutually uncorrelated***
    - *$E[a_n^* a_{n+m}] = \sigma^2+\mu^2$ for $m=0$ $E[a_n^* a_{n+m}] = \mu^2$ For $m\neq 0$*
- ***The PSD Equation***

$$s_{xx}(f) = \frac{\sigma^2}{T}\left|G(f)\right|^2 + \frac{\mu^2}{T^2}\sum_{m=-\infty}^{\infty}\left|G\left[\frac{m}{T}\right]\right|^2 \delta\left(f - \frac{m}{T}\right)$$

PSD of Some Modulation Schemes

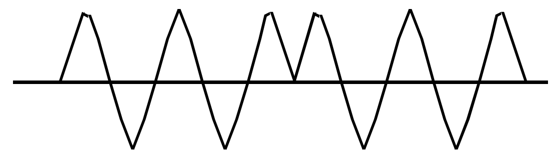


Peak to Average Power (PAP) Ratio

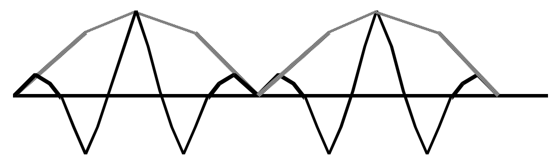
- ***Definition:***
  - ***PAP Ratio: The ratio of the peak power of the signal ENVELOPE to its average value.***
  - ***Crest factor: The ratio of the peak power of the actual (real) transmitted waveform to its average value***
- ***QPSK with square pulse shape has constant envelope***
- ***Filtering QPSK causes the signal amplitude to vary significantly***
  - ***Strongest dip occurs when signal passes through zero envelope***
    - *Instantaneous average power is reduced during this transition*
    - *Overall average power is reduced and the PAP ratio is increased*
  - ***OQPSK and $\pi/4$-QPSK reduce the PAP ratio by eliminating transitions through the origin (look at constellation plot)***

PAP cont'd

- ***Ratio of the peak power of the modulated carrier to its average power***
  - ***Ideal BPSK using square pulses.***



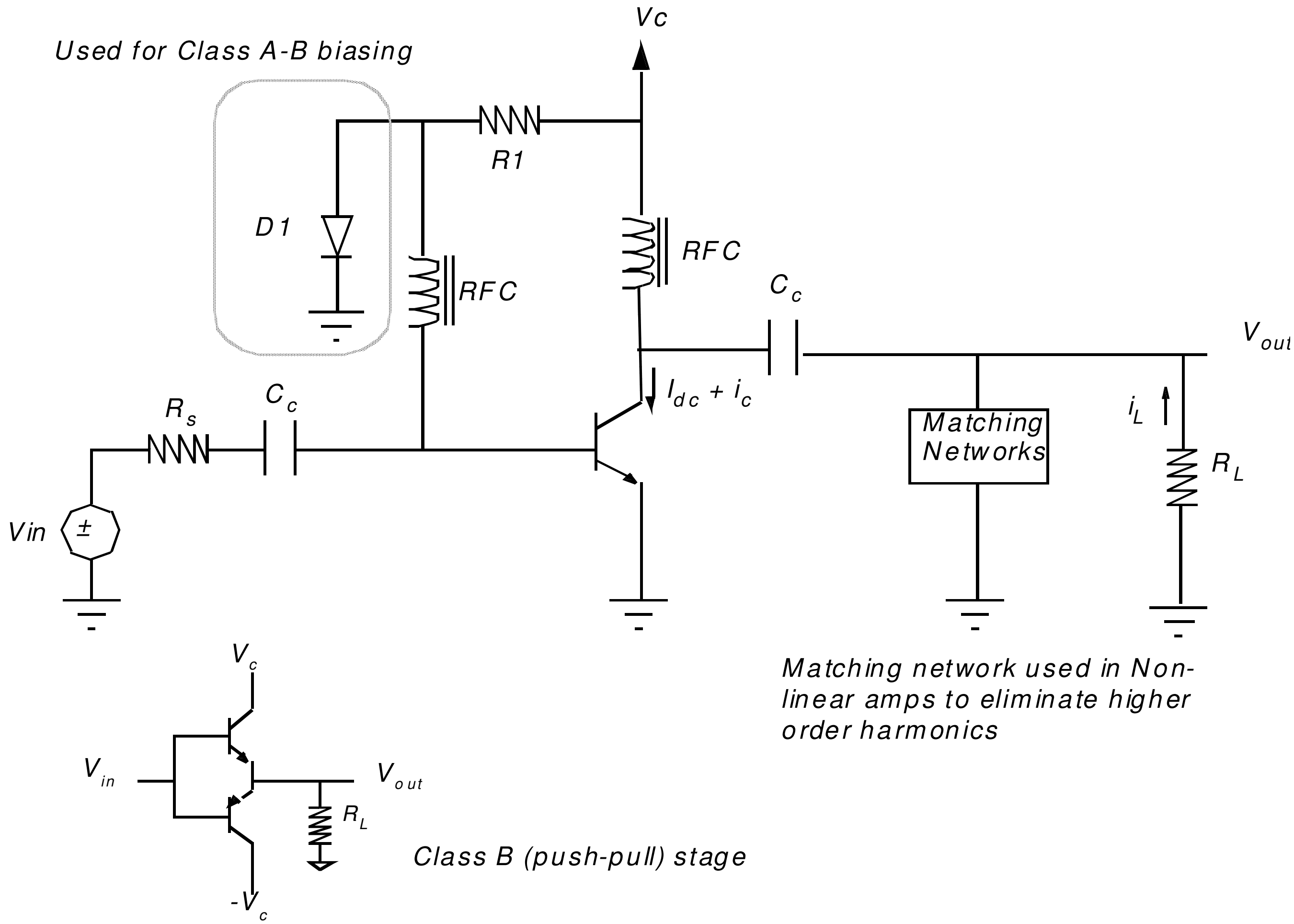
    - *Peak power = A^2*
    - *Average power = average power per period = $A^2/2$*
    - *PAP ratio is a constant for all time.*
  - ***Filtered BPSK***



    - *Peak power = A^2*
    - *Average power per carrier period = $A^2 a_n/2$, where $a_n<1$ varies from one period of the carrier to the next*
  - ***Overall signal's PAP ratio is lower than that of the ideal BPSK signal***

- ***Power amplifier wastes DC power during cycles when carrier amplitude is small.***
  - ***Lower amplifier power efficiency (ratio of DC power dissipation to output signal power)***

Power Amplifier



Matching network used in Non-linear amps to eliminate higher order harmonics

Class B (push-pull) stage

Third order Intercept Point